

Tetraquark bound states in a Bethe-Salpeter approach

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We determine the mass of tetraquark bound states from a coupled system of covariant Bethe-Salpeter equations. Similar in spirit to the quark-diquark model of the nucleon, we approximate the full four-body equation for the tetraquark by a coupled set of two-body equations with meson and diquark constituents. These are calculated from their quark and gluon substructure using a phenomenologically well-established quark-gluon interaction. For the lightest scalar tetraquark we find a mass of the order of 400 MeV and a wave function dominated by the pion-pion constituents. Both results are in agreement with a meson molecule picture for the $f_0(600)$. Our results furthermore suggest the presence of a potentially narrow all-charm tetraquark in the mass region 5 – 6 GeV.

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Introduction

The nature of the light scalar 0^{++} meson nonet is still an issue under debate. Experimentally, the lowest-lying states in this channel are very broad which makes it difficult to identify their structure and properties. While other multiplets are firmly established to be $\bar{q}q$ mesons, the lightest scalar mesons σ , κ , $a_0(980)$ and $f_0(980)$ display signatures that signal a potential strong non- $\bar{q}q$ component. Once abandoned from the particle data book, these states were reintroduced only in the last decade due to new experiments such as KLOE in the $e^-e^+ \rightarrow \pi^0\pi^0\gamma$ channel [1] or BES in the $J/\Psi \rightarrow \omega\pi^-\pi^+$ channel [2]; see also [3] for a thorough compilation.

Recently, several approaches to data analyses utilizing conventional and modified Roy equations have deduced a pole mass and width of the order of $m_\sigma \approx 450 + i280$ MeV for the $\sigma/f_0(600)$ [1, 4], with error bars below the five-percent level. Of course, these analysis do not answer the question on the nature of these states. In this respect, a number of arguments surfaced over the years which question the $\bar{q}q$ nature of light scalar mesons. In the non-relativistic quark model, scalar quarkonia are p waves, yet the 0^{++} nonet members are comparatively light and the $f_0(600)$ lies even below the 1^{--} isoscalar (s -wave) ω meson. The mass ordering within the 0^{++} nonet is puzzling as well: the lowest-lying state is the isoscalar $f_0(600)$ instead of the isotriplet $a_0(980)$, and the mass degeneracy of $a_0(980)$ and $f_0(980)$ is hard to reconcile with their different flavor content. Finally, their decay channels disagree with a $\bar{q}q$ picture: both $a_0(980)$ and $f_0(980)$ couple to $K\bar{K}$ although only the latter contains strangeness, and the broadness of the dominant decay channel $f_0(600) \rightarrow \pi\pi$ remains unexplained.

The unexpected behavior of these light scalar mesons can be resolved in a tetraquark assignment which was introduced long ago by Jaffe [5]. Here, the oddities mentioned above are naturally explained by the flavor structure of the $qq\bar{q}\bar{q}$ nonet. The mass spectrum deduced from the tetraquark nonet is inverse to that of the $q\bar{q}$ nonet, thus explaining the low mass of the isoscalar. A 0^{++} tetraquark carries zero quark orbital angular mo-

mentum [6], in agreement with the expectation that such s -wave states should be light. The putative decay channels of the tetraquark nonet agree with the observed ones, i.e., the coupling of a_0 and f_0 to $K\bar{K}$ and $\eta\pi$ is caused by their strange-quark content, while the broadness of the $f_0(600)$ is a consequence of its OZI-superalowed decay into two pions. These phenomenological arguments based on the group structure of the tetraquark flavor nonet are backed by effective theory studies and large- N_c arguments (see e.g. [7–12] and references therein) as well as recent lattice calculations [13–15] which suggest a strong $qq\bar{q}\bar{q}$ -component in the lowest-lying 0^{++} states.

In this letter we present the first results for tetraquarks in a covariant continuum approach based on the corresponding four-body equation for two quarks and two antiquarks [16]. We construct a suitable representation of this system in terms of mesons and diquark degrees of freedom. While we choose this approximation for the sole reason of its numerical simplicity compared to the full four-body equation, its feasibility is well motivated by recent results in the baryon sector. There, the analogous quark-diquark approximation to the nucleon's three-body Faddeev equation works well on the five-percent level [17–19]. The relevance of diquark components in the systematics of hadron physics has also been emphasized in Refs. [20]. It is furthermore important to note, that the meson and diquark degrees of freedom that appear in our approach are not fundamental; they are obtained dynamically from quark-(anti-)quark Bethe-Salpeter equations (BSEs) and a model for the quark-gluon interaction which has been very successful on a phenomenological level [21]. We therefore determine the properties of tetraquarks from the fundamental quark and gluon degrees of freedom of QCD.

The Bethe-Salpeter equation for tetraquarks

We start with the full four-quark Green function which satisfies the relation

$$G = G_0 + G_0 T G_0. \quad (1)$$

Here, G_0 is the product of four dressed quark propagators and T represents the connected and fully amputated four-

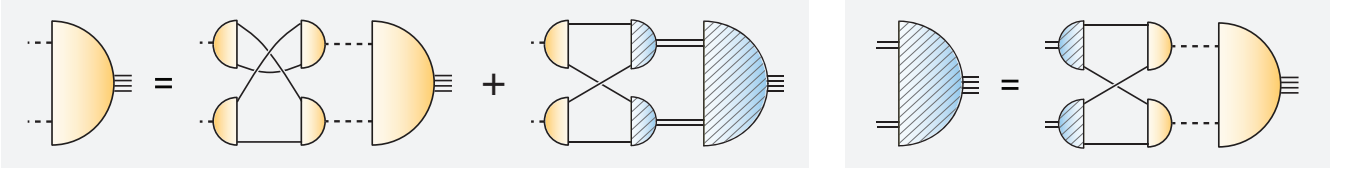


FIG. 1: Tetraquark BSE in the meson-meson/antidiquark-diquark picture. The hatched amplitudes involve diquark quantities; the remaining ones are of mesonic nature. Single (double, dashed) lines are dressed quark (diquark, meson) propagators.

quark scattering matrix. In our symbolic notation the multiplications in Eq. (1) represent four-dimensional integrations over the appropriate number of momenta and all indices are left implicit. Tetraquark bound states with mass M appear as poles in the scattering matrix

$$T \xrightarrow{P^2 = -M^2} \frac{\Psi \bar{\Psi}}{P^2 + M^2} \quad (2)$$

and thereby define the tetraquark's covariant bound-state amplitude Ψ , with $\bar{\Psi}$ being its charge conjugate and P the total momentum.

The Green function satisfies a Dyson equation which relates it to the four-body interaction kernel K via

$$G = G_0 + G_0 K G \Leftrightarrow T = K + K G_0 T. \quad (3)$$

Substituting Eq. (2) into (3) and projecting onto the singular part, one arrives at a homogeneous bound-state equation for the tetraquark amplitude:

$$\Psi = K G_0 \Psi. \quad (4)$$

This equation can be solved once the four-body kernel K is known. The kernel contains 2PI, 3PI and 4PI contributions [16]; in the following we adopt the successful strategy of Ref. [18] and neglect the latter two.

To simplify the notation, we suppress the quark propagators by replacing $G_0^{-1} G \rightarrow G$, $T G_0 \rightarrow T$ and $K G_0 \rightarrow K$, so that Eqs. (1), (3) and (4) become

$$G = 1 + T = 1 + K G, \quad T = K (1 + T), \quad \Psi = K \Psi.$$

The remaining part of the kernel that involves only two-body correlations can be written as the sum of three terms:

$$K = \sum_{aa'} K_{aa'}, \quad (5)$$

where a and a' denote qq , $\bar{q}\bar{q}$ or $q\bar{q}$ pairs, so that aa' is one of the three combinations (12)(34), (13)(24) or (14)(23). Therefore, $K_{aa'}$ describes the component of the four-body kernel where all interactions are switched off except those within the pairs a and a' .

The absence of residual color forces between widely separated clusters, potentially generated by the permuted interactions in Eq. (5), amounts to the condition

$$G_{aa'} = G_a G_{a'}. \quad (6)$$

Here, $G_{aa'} = 1 + T_{aa'}$ is the four-body Green function obtained from the kernel $K_{aa'}$, whose scattering matrix satisfies

$$T_{aa'} = K_{aa'} (1 + T_{aa'}), \quad (7)$$

and $G_a = 1 + T_a$, $G_{a'} = 1 + T_{a'}$ are the two-body Green functions for the individual pairs a and a' , with

$$T_a = K_a (1 + T_a), \quad T_{a'} = K_{a'} (1 + T_{a'}). \quad (8)$$

The separability of the Green function $G_{aa'}$ imposes the following structure on the kernel $K_{aa'}$ [16]:

$$K_{aa'} = K_a + K_{a'} - K_a K_{a'}, \quad (9)$$

where K_a and $K_{a'}$ are now elementary qq , $\bar{q}\bar{q}$ or $q\bar{q}$ kernels. The relations (7–9) yield the scattering matrix

$$T_{aa'} = T_a + T_{a'} + T_a T_{a'}, \quad (10)$$

from which Eq. (6) can be readily verified.

In principle, the tetraquark bound-state equation (4) with the kernel of Eqs. (5) and (9) can be solved with the techniques used in Ref. [18] for the three-body equation. In practice, however, this is a very demanding task in terms of computation power. While we strive to attack this problem in the future, for now we resort to a further, simplifying approximation in the spirit of the nucleon's Faddeev equation and its reduction to a quark-diquark picture [17]. In analogy to the three-body case, we define Faddeev amplitudes $\Psi_{aa'}$ via

$$K_{aa'} \Psi =: \Psi_{aa'} \Rightarrow \Psi = \sum_{aa'} \Psi_{aa'}. \quad (11)$$

Upon projecting Eq. (7) onto the bound-state amplitude Ψ , one obtains Faddeev-Yakubovsky type equations [22] for the amplitudes $\Psi_{aa'}$:

$$\Psi_{aa'} = T_{aa'} (\Psi_{bb'} + \Psi_{cc'}), \quad aa' \neq bb' \neq cc', \quad (12)$$

where $T_{aa'}$ is constructed from two-body scattering matrices according to Eq. (10).

Except for the omission of genuine three- and four-body correlations, Eq. (12) is still exact. Its reduction to a two-body problem proceeds by assuming that the two-body T -matrices are dominated by meson and diquark pole contributions,

$$T_a(q_1, q_2, Q) = -\Gamma_a(q_1, Q) D_a(Q) \bar{\Gamma}_a(q_2, Q), \quad (13)$$

and that the internal spin-momentum structure of the Faddeev amplitudes factorizes:

$$\Psi_{aa'}(p, q, q', P) = \Gamma_a(q, Q) D_a(Q) \times \bar{\Gamma}_{a'}(q', Q') D_{a'}(Q') \Phi_{aa'}(p, P). \quad (14)$$

Without loss of generality we can assign the labels 1, 2 to the quarks and 3, 4 to the antiquarks. Then, for $aa' = (12)(34)$, Γ_a and $\bar{\Gamma}_{a'}$ describe diquark and antidiquark bound-state amplitudes and $D_a, D_{a'}$ their respective propagators, whereas in the case of $aa' = (13)(24)$ or $(14)(23)$ the involved objects are of mesonic nature. Quantities with a bar indicate charge-conjugated amplitudes. P is the total tetraquark momentum and p the relative momentum between its respective constituents. The separated internal momenta q, q' correspond to the relative momenta of the (anti-)diquarks and mesons. Their total momenta Q, Q' can take arbitrary values and thus an off-shell description for the meson and diquark amplitudes is necessary.

Combining Eqs. (12–14) and furthermore neglecting the single-interaction contributions T_a and $T_{a'}$ from Eq. (10) yields a coupled diquark-antidiquark/meson-meson BSE which is depicted in Fig. 1. It describes an effective interaction between two mesons, or between a diquark and an antidiquark, via quark exchange. We take into account the mesons and diquarks with lowest mass, i.e., the pseudoscalar-meson and scalar-diquark channels. Constituents with other quantum numbers are certainly possible; however, since the corresponding calculations are very expensive in terms of CPU time we postpone their inclusion to subsequent work. Moreover, we only investigate the lowest-lying tetraquark with quantum number $J^P = 0^+$. The resulting diquark-antidiquark and meson-meson contributions to the tetraquark amplitude,

$$\begin{aligned} \Phi_D(p, P) &:= \Phi_{(12)(34)}, \\ \Phi_M(p, P) &:= \Phi_{(13)(24)} = -\Phi_{(14)(23)}, \end{aligned} \quad (15)$$

are flavor and color singlets and Lorentz scalars.

It is noteworthy that our framework, Fig. 1, does not permit a pure diquark-antidiquark state in isolation; it can only occur in combination with meson-meson interactions. On the other hand, both equations can be merged to a single mesonic equation where diquarks appear only internally. Thus, one may view the resulting tetraquark bound state as a meson molecule with diquark-antidiquark admixture to its kernel. We expect this diquark admixture to be especially important for tetraquarks with masses larger than the sum of their meson constituents.

Mesons and diquarks from quark and glue

In order to solve the tetraquark BSE of Fig. 1, we need to determine the (on-shell) masses and amplitudes of the meson and diquark building blocks as well as a suitable continuation off their mass shells. For diquarks, this problem has been dealt with already within the quark-diquark approach to the baryon three-body problem [17]. The corresponding techniques are well developed and their reliability can be judged from the good

agreement of nucleon and Δ masses in the quark-diquark picture with results from the corresponding three-body problem [18, 28]. We therefore adopt this framework also for the diquark and meson amplitudes that appear in our setup. The technical details of these types of calculations have been described in many works, see e.g. [21, 23, 24] for reviews, thus we only give a short summary here.

The starting point is the Dyson-Schwinger equation for the dressed quark propagator,

$$S_{\alpha\beta}^{-1}(p) = Z_2 (i\not{p} + m_0)_{\alpha\beta} + \int_q \mathcal{K}_{\alpha\alpha'\beta'\beta} S_{\alpha'\beta'}(q), \quad (16)$$

with wave-function renormalization constant Z_2 and bare quark mass m_0 . The exact interaction kernel $\mathcal{K}_{\alpha\alpha'\beta'\beta}$ contains the dressed gluon propagator as well as one bare and one dressed quark-gluon vertex. The Greek subscripts refer to color, flavor and Dirac structure. In the rainbow-ladder approximation that we adopt here the kernel can be written as

$$\mathcal{K}_{\alpha\alpha'\beta'\beta} = Z_2^2 \frac{4\pi\alpha(k^2)}{k^2} T_k^{\mu\nu} \gamma_{\alpha\alpha'}^\mu \gamma_{\beta\beta'}^\nu, \quad (17)$$

where $T_k^{\mu\nu} = \delta^{\mu\nu} - k^\mu k^\nu / k^2$ is a transverse projector with respect to the gluon momentum k . Eq. (17) describes an iterated dressed-gluon exchange between quark and antiquark that retains only the vector part $\sim \gamma^\mu$ of the quark-gluon vertex. Its non-perturbative dressing, together with the one for the gluon propagator, is absorbed into an effective coupling $\alpha(k^2)$ which is taken from Ref. [18, 25].

Chiral symmetry and the associated axial-vector Ward-Takahashi identity demand the kernel \mathcal{K} to appear in the corresponding BSEs as well. The meson BSE is given by

$$\Gamma_{\alpha\beta}(p, P) = \int_q K_{\alpha\alpha'\beta'\beta} \{S(q_+) \Gamma(q, P) S(q_-)\}_{\alpha'\beta'}, \quad (18)$$

with the Bethe-Salpeter amplitude $\Gamma(p, P)$ depending on total and relative momenta of the quark and anti-quark constituents, and $q_\pm = q \pm P/2$. The corresponding equation for diquarks is obtained by the substitution of an antiquark with a quark leg. Both meson and diquark amplitudes contain four different Dirac structures which we compute explicitly.

With a given effective coupling, one determines the quark propagator in the complex momentum plane and subsequently solves the meson and diquark BSEs. According to the techniques developed in Refs. [17, 24], the resulting onshell wave functions are analytically continued to offshell momenta and the effective meson and diquark propagators are computed from their T -matrix relations. Finally, all building blocks are put together in the tetraquark BSE. Our numerical techniques used to solve this BSE are only slightly non-standard and will be described in detail elsewhere.

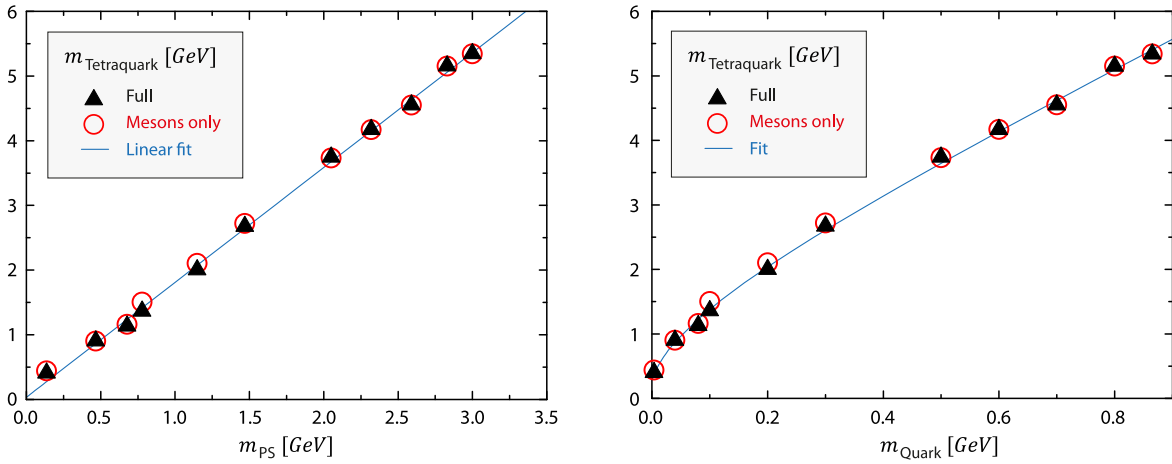


FIG. 2: Mass of the 0^{++} tetraquark as a function of the pseudoscalar-meson mass (left) and the quark mass (right panel).

Results and discussion

Our result for the mass of the up/down 0^{++} tetraquark state as a function of the pseudoscalar-meson mass is shown in the left panel of Fig. 2, together with a calculation that only includes the meson-molecule component of the tetraquark. Clearly, the meson-meson component dominates. Except for the chiral region, the overall dependence of the tetraquark mass upon the pseudoscalar-meson mass is linear within numerical errors, as can be seen from the comparison with the linear fit included in the plot. The reason for this behavior is even more clear from the right panel of Fig. 2, where we plot the tetraquark mass as a function of the quark mass. The line represents a fit to the data including a constant, a square root and a linear term. Apart from the constant term this is the typical behavior of a Goldstone boson. The tetraquark thus grossly inherits the mass behavior of its dominating pion-molecule constituents, with deviations generated from their interactions via quark exchange.

One of the main results of our present work is the value for the u/d tetraquark at the physical point, i.e. the leftmost points in Fig. 2, where $m_{\text{PS}} = m_{\pi}$. We obtain:

$$m_{\text{Tetraquark}}^{u/d}(0^{++}) = 403 \text{ MeV}, \quad (19)$$

with an estimated numerical error of ten percent. This value is only somewhat lower than the real part of the mass of the $\sigma/f_0(600)$, $m_{\sigma} \approx 450 + i280 \text{ MeV}$ determined recently from experiment using Roy equations [1, 4]. Our value for the mass of the scalar tetraquark should also be compared with the corresponding one for an ordinary quark-antiquark scalar bound state which may mix with the tetraquark components. In our rainbow-ladder approximation such a state has a mass of $m_{q\bar{q}}(0^{++}) = 665 \text{ MeV}$. It is well known that corrections beyond rainbow-ladder increase this value into the 1 GeV range [26, 27], whereas the pion mass is protected. Since our tetraquark is dominated by its meson-molecule nature, we therefore expect it to be stable against corrections beyond rainbow-ladder, while at the same time the mass

splitting between the tetraquark and the quark-antiquark scalar will increase. Consequently, our results suggest to identify the physical lowest-lying scalar state to be dominated by a strong tetraquark component, which is in turn dominated by pion molecule contributions. Due to the (Pseudo-) Goldstone nature of the pion constituents, our result provides a ready and natural explanation for the small mass and the large decay width of the $\sigma/f_0(600)$.

In the strange quark region at about $m_{\text{Quark}} = 80 \text{ MeV}$ we also observe an all-strange tetraquark bound state at roughly $m_{\text{Tetraquark}}^s(0^{++}) = 1.2 \text{ GeV}$. Certainly this state will mix with its pure $s\bar{s}$ counterpart as well as the lowest lying scalar glueball making an identification with $f_0(1500)$ or $f_0(1710)$ not possible without further studies.

It is furthermore interesting to speculate about the existence of an all-charm tetraquark state. Because of its flavor-structure in our meson-diquark picture, such a state would be a mixture between a meson and an axialvector-diquark component. Since already the scalar-diquark contribution is very small, we expect the axialvector component to be completely suppressed due to its larger mass. This leaves only the dominant meson-molecule part. In Fig. 2 the largest pseudoscalar-meson mass corresponds to a quark mass in the charm region. We therefore read off the mass of an all-charm scalar tetraquark state to be at

$$m_{\text{Tetraquark}}^c(0^{++}) = 5.3 \pm (0.5) \text{ GeV}, \quad (20)$$

where the error is a guess based on our numerical and systematic uncertainties. This mass is considerably lower than the 6.2 GeV obtained in simple model calculations [29, 30]. It is also much lower than the η_c threshold. Potential decay channels into D mesons and pairs of light mesons necessarily involve internal gluon lines. The resulting decay width may therefore be rather small.

Further results for tetraquark states with unequal mass constituents are numerically more demanding than the ones presented here and will only be available for a future publication.

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